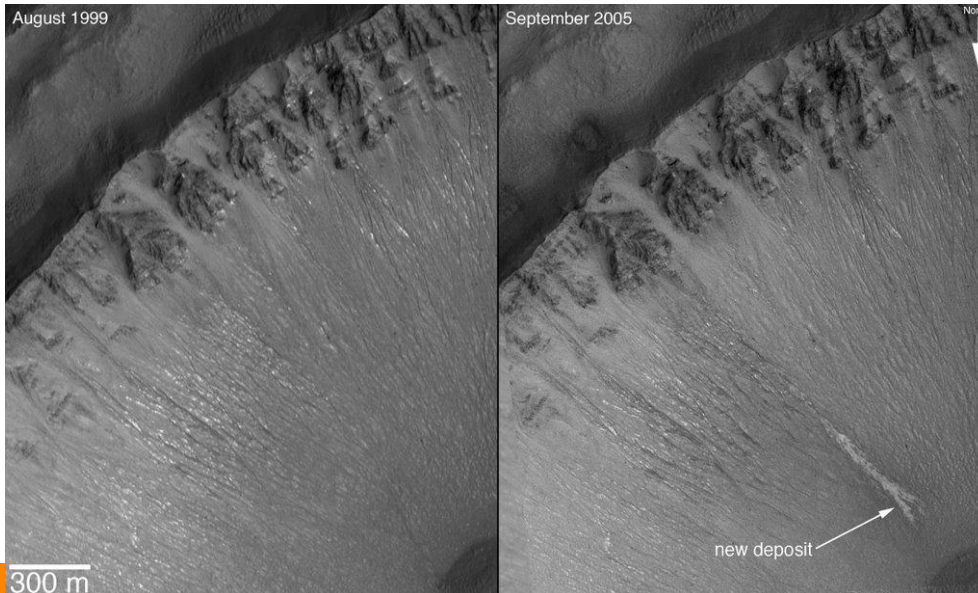


Geodynamics: The Physics and Mathematics of Terrestrial and Extra-Terrestrial Processes

Stuart R. Clark

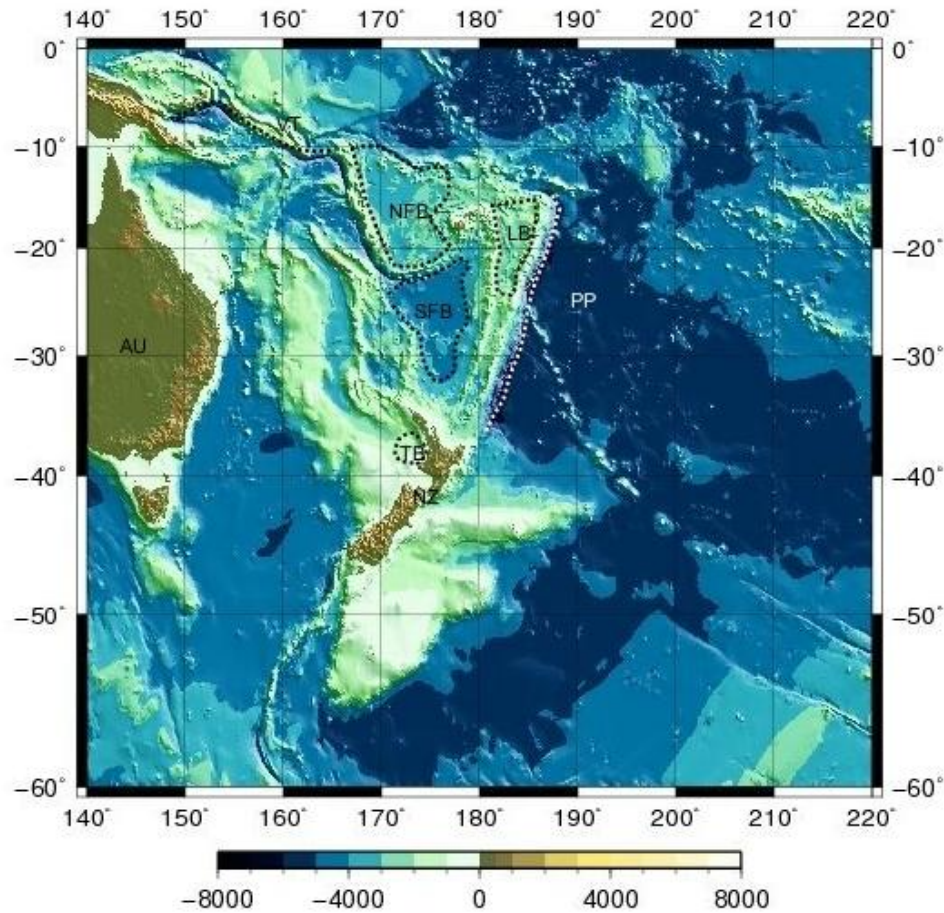
Computational Geosciences
Project
Center for Biomedical Computing



Badlands, Mojave National Preserve (courtesy USGS)

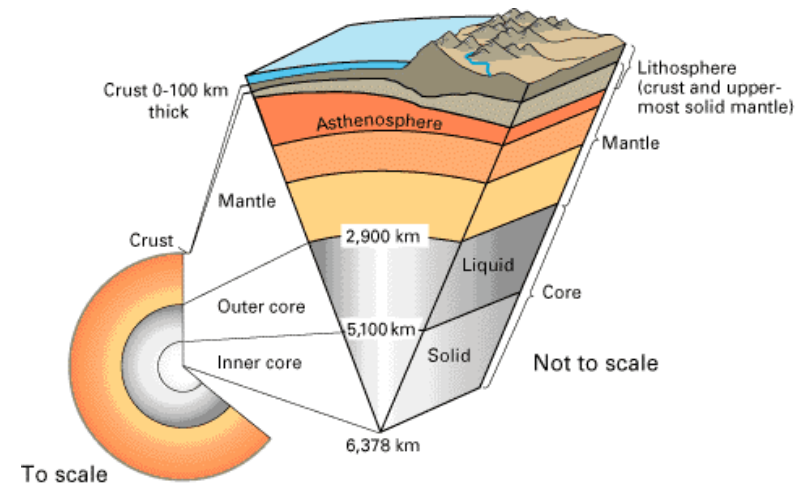
Mars' Centauri Montes crater
(courtesy NASA/JPL/Malin Space Science Systems)

Primary goal: Understanding the present state of the earth or planet together and the history of the structures



Navier-Stokes Equation governs mantle flow on geological time-scales

$$\rho \frac{D\mathbf{v}}{Dt} = -\nabla p + \nabla \cdot \mathbb{T} + \mathbf{f}$$



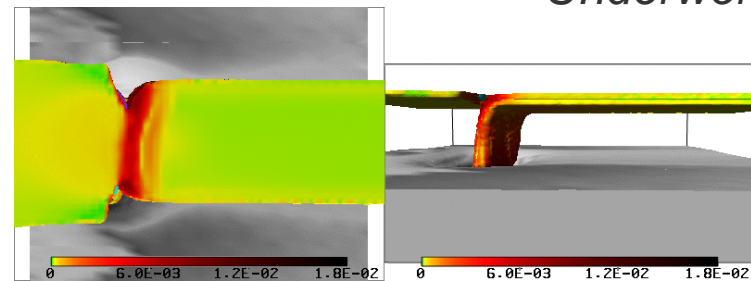
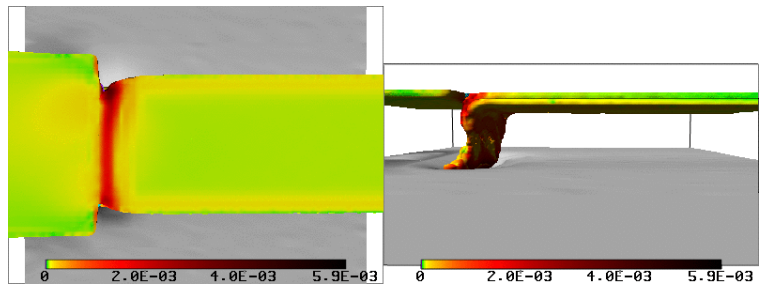
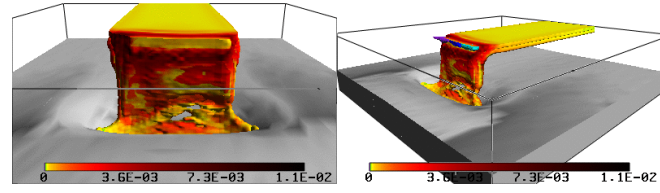
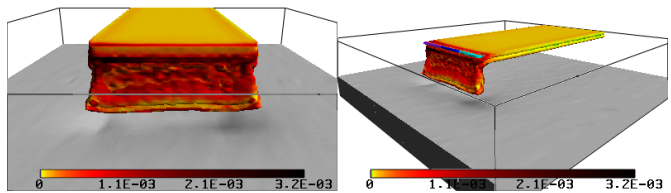
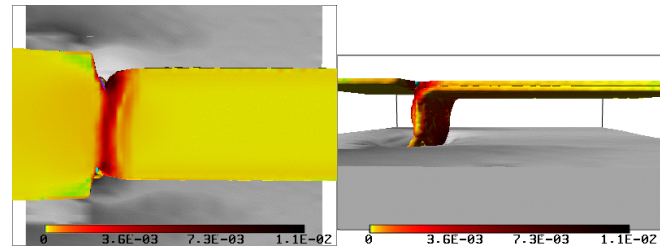
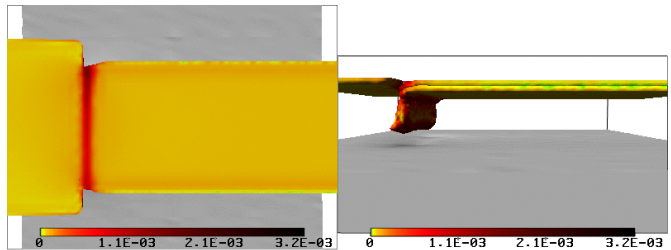
Just as Navier-Stokes can be applied to flow in a pipe or a blood vessel, so it can be applied to the “flow” of rocks!

Turbulence can safely be neglected in the mantle, as the Reynolds number is extremely small

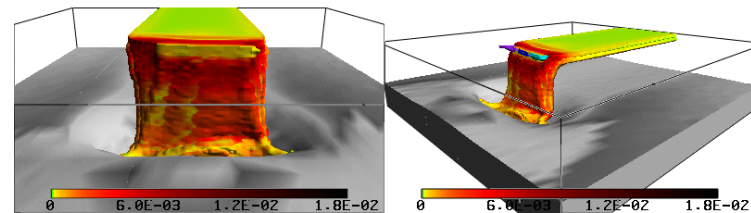
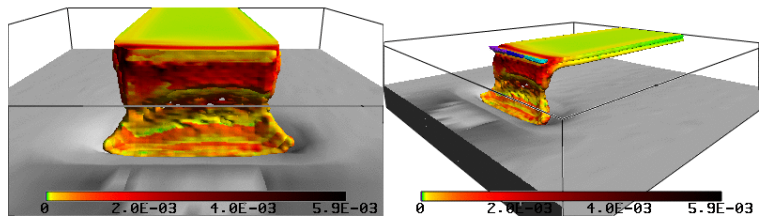
$$\begin{aligned} Re &= \frac{\rho v D}{\eta} \\ &= \frac{3.8 \times 10^3 \text{ kgm}^{-3} \times 5 \times 10^{-9} \text{ ms}^{-1} \times 2.8 \times 10^6 \text{ m}}{2 \times 10^{21}} \\ &= 2.7 \times 10^{-20} \end{aligned}$$

v is a typical velocity for the fluid, D is a typical length scale for the flow, η is the dynamic viscosity

Models with visco-plastic rheologies can give insights into the nature of subduction zones

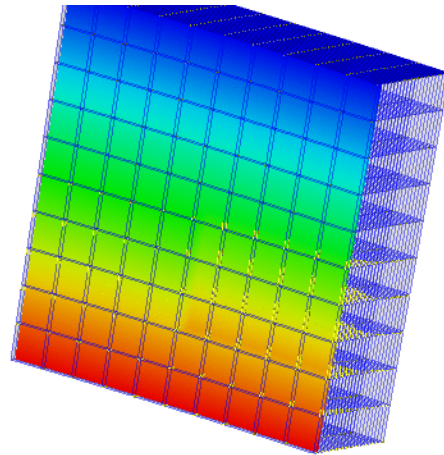


Underworld: MPICH2



The prime motivation for these models is to understand the present day structure and its history

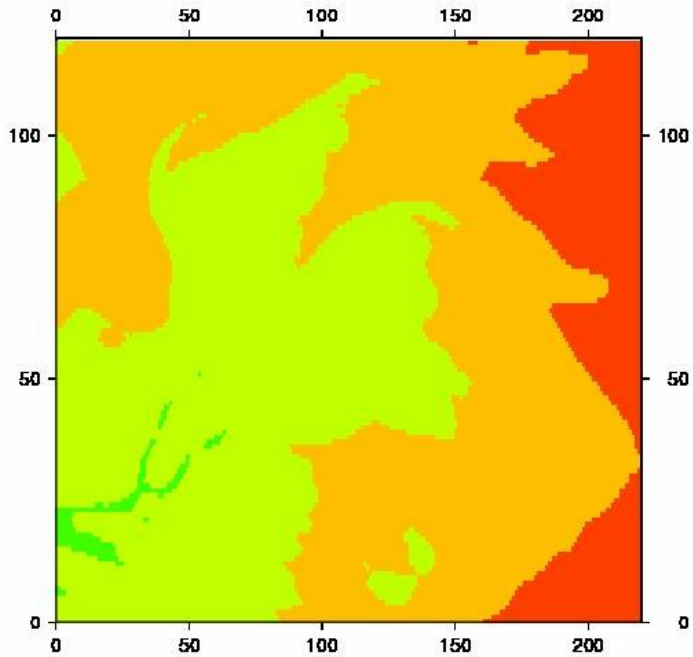
$$\frac{\partial T}{\partial t} = -\vec{u} \cdot \nabla T + \nabla^2 T$$



CitcomS: OpenMPI

Advection-Diffusion of temperature

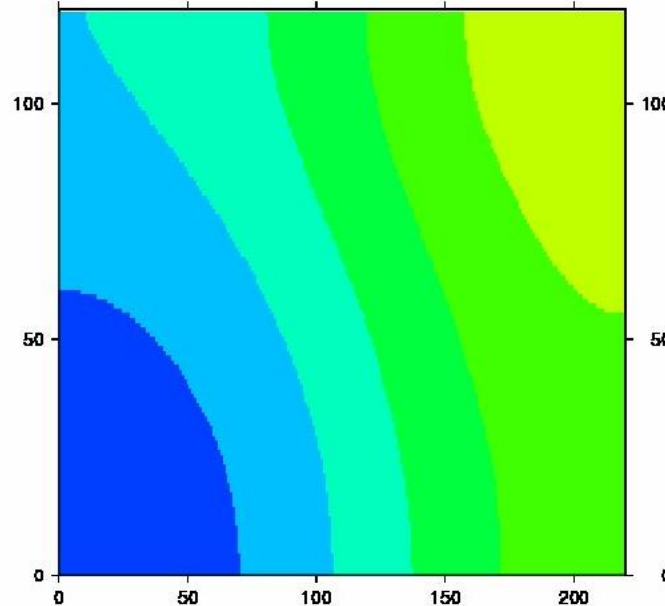
Collaboration with Xing Cai & Wenjie Wei: A diffusion-based MPI code for surface processes



Initial Height / Final Sand Fraction



Endurance Crater,
Mars



Height

Convection is the dominant mode of energy transport in the mantle,.

$$Ra = \frac{\rho g \alpha \Delta T D^3}{\eta \kappa}$$

ρ is the density, g is the acceleration due to gravity, α is the thermal expansion coefficient, η is the dynamic viscosity, κ is the thermal diffusivity

$$= \frac{3.8 \times 10^3 \text{ kgm}^{-3} \times 10 \text{ ms}^{-2} \times 1.4 \times 10^{-5} \times 1400^\circ \text{ K} \times (2.8 \times 10^6 \text{ m})^3}{2 \times 10^{21} \times 2.5 \times 10^{-6}}$$
$$= 3.34 \times 10^6$$

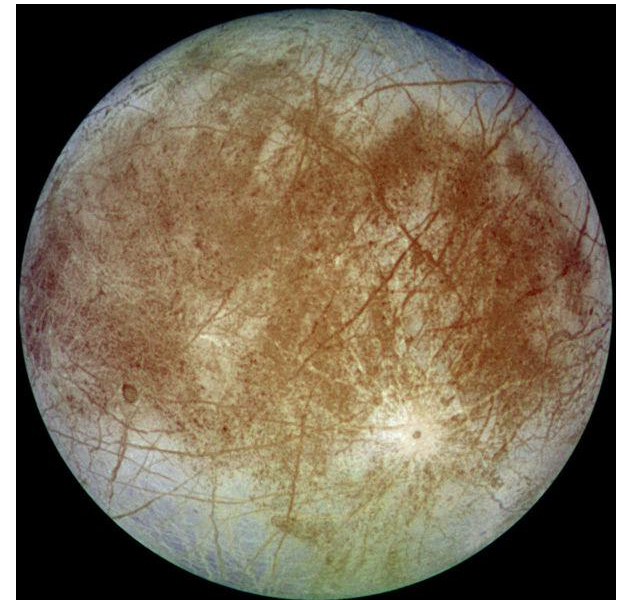
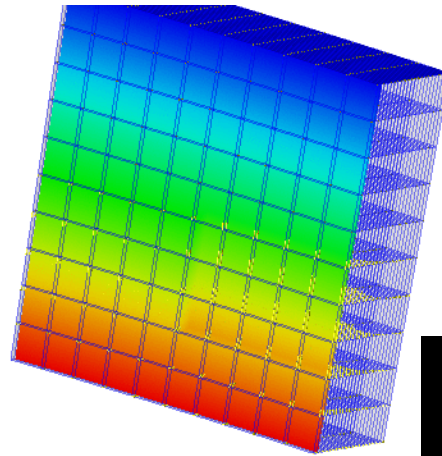
The Rayleigh number far exceeds the critical Rayleigh number, approximately 1000, implying that convection dominates conduction.

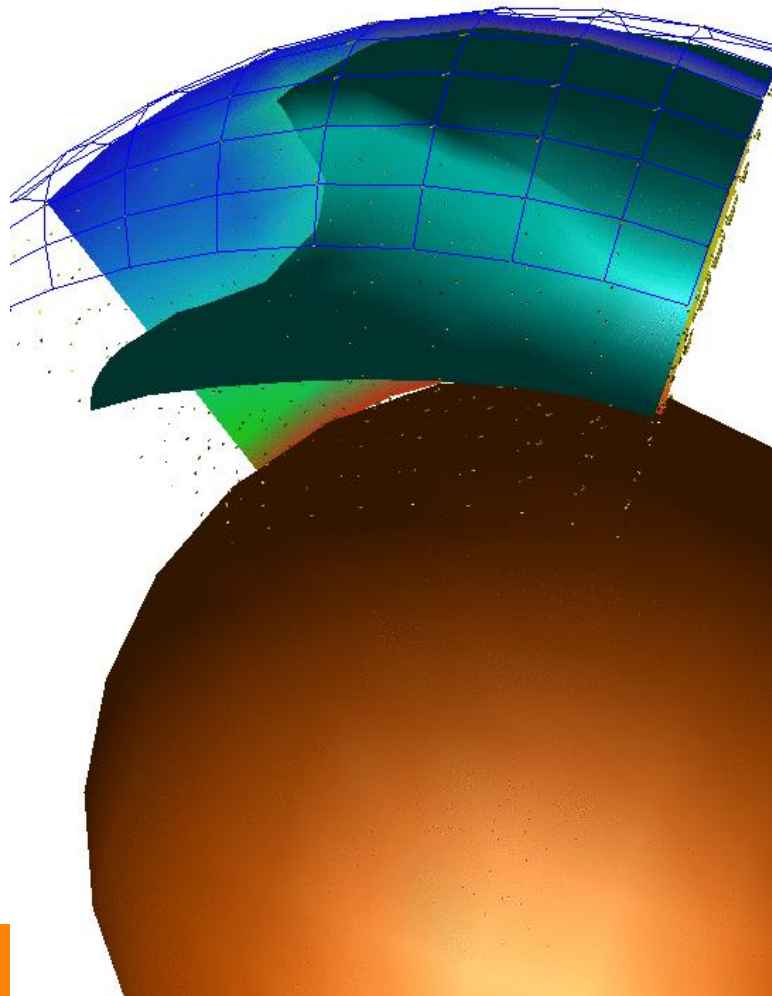
Physical parameters for different cases reveal differences in behaviour

Physical Parameter	Units	Earth	Mars	Europa	Body
Length Scale	m	2.82E+06	1.90E+06	1.50E+04	1.00E-03
Typical Velocity	ms ⁻¹	5.00E-09	5.00E-09	5.00E-08	3.00E-01
Temperature Change	K	1.40E+03	1.45E+03	1.80E+02	
Density	kgm ⁻³	3.80E+03	3.90E+03	9.26E+02	1.00E+03
Dynamic Viscosity	kgm ⁻¹ s ⁻¹	2.00E+21	2.00E+21	1.00E+13	1.50E-03
Kinematic Viscosity	m ² s ⁻¹	5.26E+17	5.13E+17	1.08E+10	1.50E-06
Reynolds Number	-	2.68E-20	1.85E-20	6.94E-14	2.00E+02
Thermal Conductivity	Wm ⁻¹ K ⁻¹	1.50E+01	1.50E+01	3.48E+00	
Thermal Diffusivity	m ² s ⁻¹	2.50E-06	2.50E-06	1.00E-06	
Expansion Coefficient	K ⁻¹	1.40E-05	1.40E-05	1.65E-04	
Rayleigh Number	-	3.34E+06	1.09E+06	9.28E+07	

Surface structures on Europa may be predominately caused by convection of a ice shell

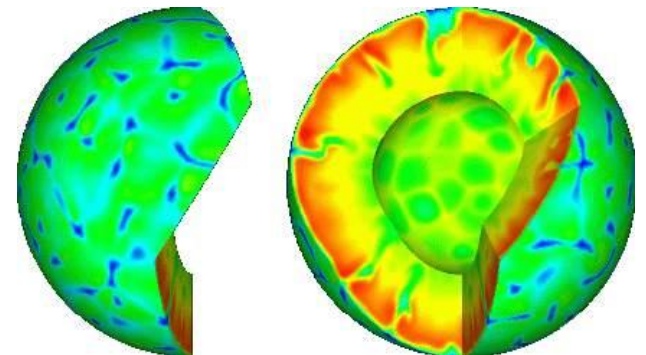
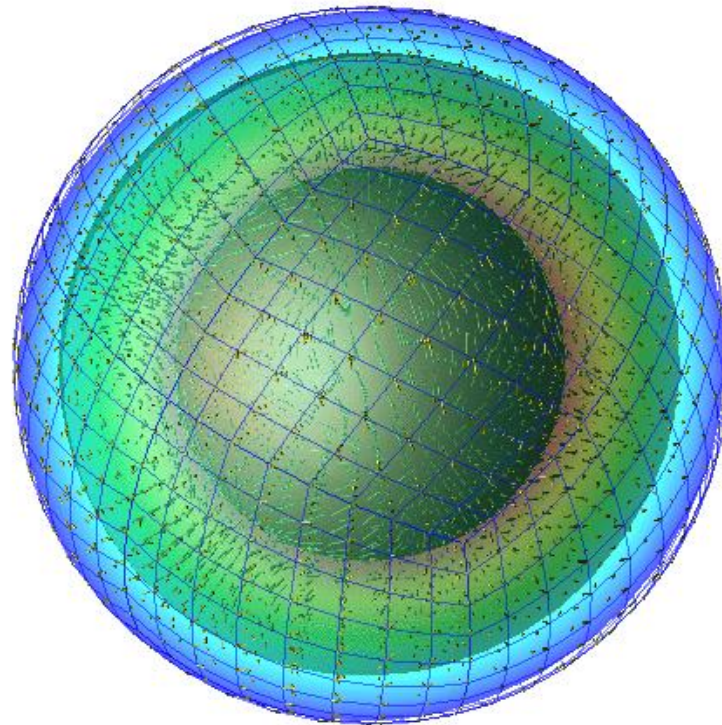
- FE Mesh
- Conjugate Gradient Solver
- 65x33x33 nodes
- ~71000 grid points
- 16 processors
- 300 timesteps
- Total flow for 192 kyrs
- Each time step shown every 32 kyrs
- Each calculation step <1 kyrs





- Multigrid Solver
- Full earth: 12 spherical caps
- 9x9x9 nodes in each cap
- 8800 grid points
- 2x2x2 processors for each cap
- 96 processors
- 1000 timesteps
- About 2Gyrs of evolution
- Time steps every 2 Myrs
- Images generated from snapshots at 200 Myrs

Large convection cells develop into long-lived convection channels or plumes



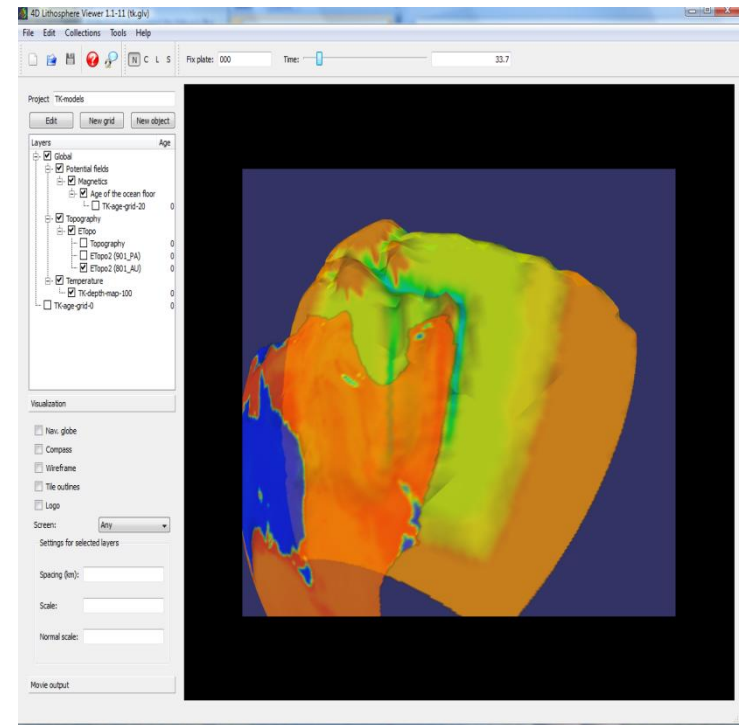
From LMU, Munich

It makes sense that Computational Geosciences is in the CBC!

We have a lot in common with CBC
groups:

- Finite Element Simulations
- Solving Navier-Stokes Equations
- Solving Laplace & similar diffusion Equations
- Parallel Computing
- Visualisation
- Inverse methods

We are hiring: so if you want a
switch...



4DLM and CitcomS (4DLM courtesy of
Kalkulo and StatoilHydro)

