

# BLOWUP OF EULER SOLUTIONS: FLOW SEPARATION

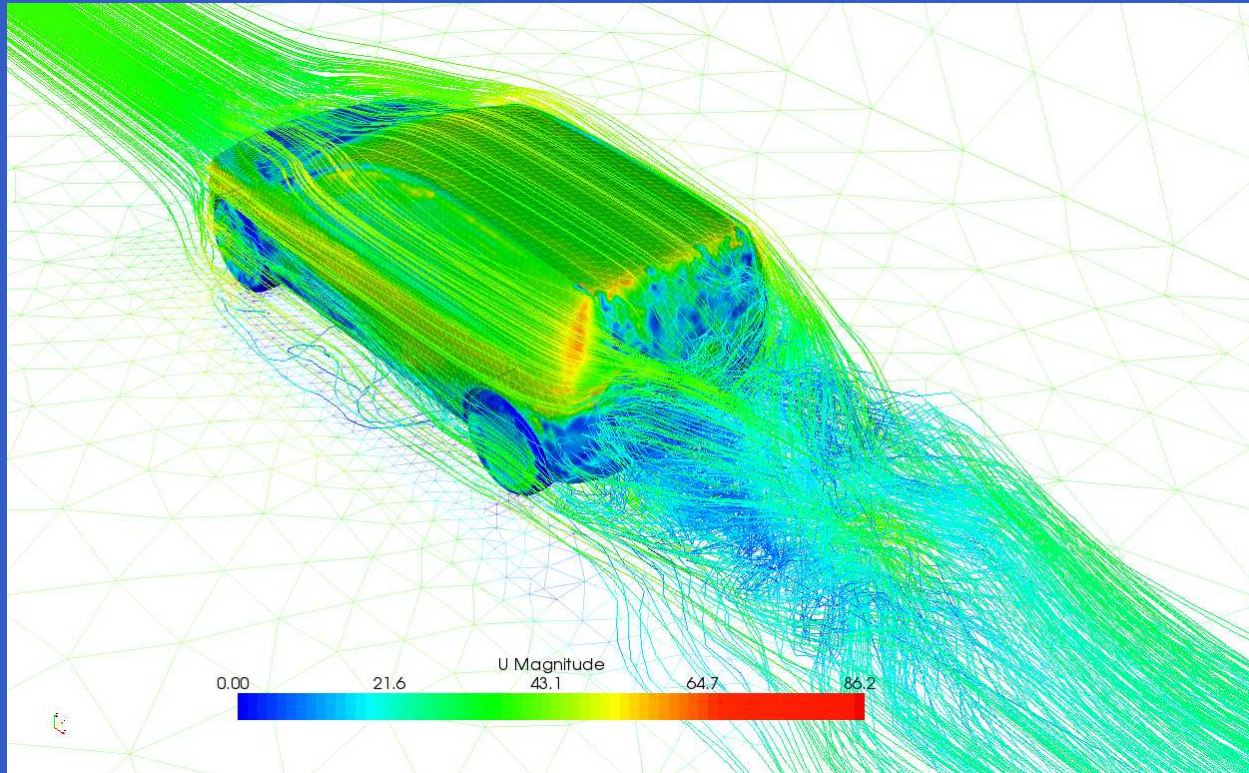
Johan Hoffman and Claes Johnson

[www.bodysoulmath.org](http://www.bodysoulmath.org), [www.fenics.org](http://www.fenics.org), [www.csc.kth.se/cgjoh](http://www.csc.kth.se/cgjoh)

# CLAY MILLENNIUM PROBLEM

- NAVIER-STOKES/EULER (INCOMP)
- (I) EXISTENCE SMOOTH SOL ALL DATA?
- or
- (II) BLOWUP FOR SPECIFIC DATA?
- ALL DATA: PROOF(I) ANALYTICAL!
- SPECIFIC DATA: PROOF (II) COMP???
- ONE MILLION DOLLAR!
- D'ALEMBERT'S PARADOX 1752

# VOLVO CAR: BLOWUP EULER



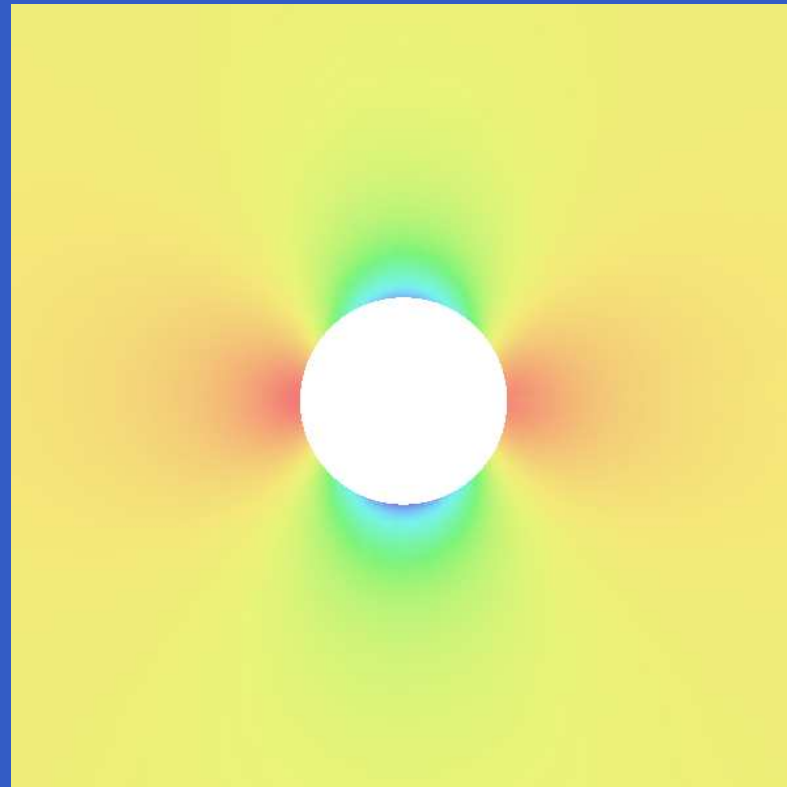
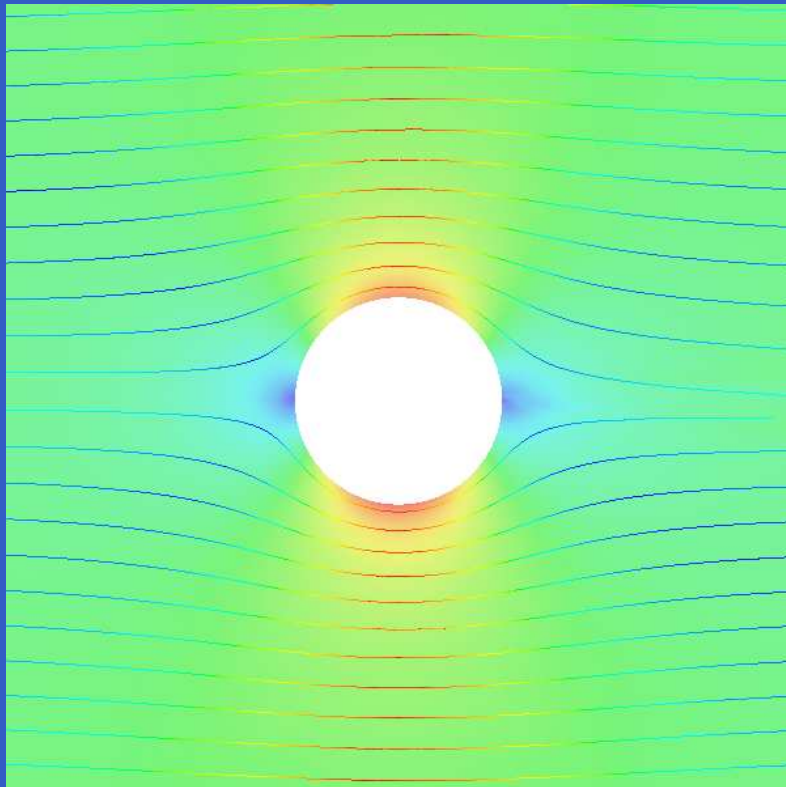
- DRAG  $\approx 0.33$ , SLIP BC
- TURBULENT INCOMPRESSIBLE EULER

# IMPOSSIBLE – POSSIBLE

- MOIN: IMPOSSIBLE:  $> 10^{16}$  MESH POINTS
- HENNINGSON: WILL TAKE 50 YEARS
- DAVIDSON: IMPOSSIBLE
- BUT POSSIBLE!! WITH  $10^6$  MESH POINTS

# POTENTIAL SOL DRAG = 0

- HOW TO SEPARATE?
- WITH NON-ZERO VELOCITY?



# SMOOTH??

- SMOOTH:
- SMALL EFFECT OF REGULARIZATION
- LARGE EFFECT OF REG = TURBULENT = BLOWUP

# FINITE MESH SIZE COMPUTATION

- BLOWUP = TURBULENCE!
- ON FINITE MESH SIZE  $H$  COMPUTATION!
- PROVES BLOWUP FOR ALL  $h < H$ ?
- TURB  $RE_H = 1/H$
- $\Rightarrow$  TURB FOR ALL  $RE > RE_H$ ?!
- CF BUCKLING

# COMPARE:

- LAMINAR FOR  $H$  or  $RE_H$
- $\Rightarrow$  LAMINAR ALL  $h < H, RE > RE_H$ ?
- FALSE!

# MORAL:

- CAN DETECT NON-SMOOTH ON FINITE MESH
- CANNOT DETECT SMOOTH ON FINITE MESH
- CLAY PROBLEM: DETECT NON-SMOOTH!!
- POSSIBLE ON FINITE MESH!
- DICHOTOMY: LAMINAR/SMOOTH – TURBULENT/NON-SMOOTH
- CF BURGERS: SMOOTH – SHOCKS

# WELLPOSEDNESS

- HADAMARD 1902
- SMALL PERTURBATIONS
- $\Rightarrow$  SMALL EFFECT
- ON OUTPUT
- ONLY WELLPOSED MEANINGFUL
- NOT WELLPOSED NOT MEANINGFUL!!
- $\nu > 0$  SAME AS  $\nu \geq 0$
- EULER INCLUDED IN CLAY NS PROBLEM!

# EXACT SOL from APPROXIMATE

- IS THERE  $x: D(x) = d$ ?
- $D(X) = d + R(X)$
- $2S|R(X)| < TOL$
- $S = |D'(X)^{-1}|$
- $S|D'(y) - D'(X)| < \frac{1}{2}$  for  $|x - X| < TOL$
- THEN  $D(x) = d$  with  $|x - X| < TOL$
- $|x - X| \leq 2S|R(X) - R(x)|$
- CONTRACTION MAP  $x \rightarrow x - D'(X)^{-1}D(x)$

# WELLPOSEDNESS

- $D(x) = d$  “EXACT” SOL  $x$
- SUBJECT to PERTURBATION  $R$
- $D(X) = d + R$
- $X$  EXACT SOL of PERTURBED DATA
- $X$  “AS GOOD” AS  $x$  IF
- $S = D'(X)^{-1}$  MODERATE SIZE!!
- TEST: WELLPOSEDNESS of COMPUTED  $X$
- $X$  REPRESENTATIVE SOLUTION

# OUTPUT WELLPOSEDNESS

- $M(x)$  OUTPUT FUNCTIONAL WEIGHT  $\psi$
- $M(X)$  WELLPOSED IF  $S$  MODERATE SIZE
- $|M(x) - M(X)| \leq S \|R(x) - R(X)\|_{-1}$
- $\|\cdot\|_{-1}$  WEAK NORM
- $S = \|\varphi\|_1$
- $D'(X)^\top \varphi = \psi$

# EQ WITHOUT EXACT SOLUTIONS

- EULERS EQUATIONS IN FLUID MECH
- WELLPOS EXACT SOL DO NOT EXIST
- WELLPOS COMP SOLUTIONS DO EXIST
- APPROX OF NON-EXISTING EXACT SOL
- SIMULATIONS OF NON-EXISTING REALITY

■ HYPERREALITY

# EQ WITHOUT EXACT SOLUTIONS

- SCHRÖDINGER'S
- KOHN NOBEL PRIZE 1998
- $\mathbb{R}^{3N}$  WAVE FUNCTION DOES NOT EXIST IF  $N \geq 100$ .
- SUPERPOSITION-SUPERSTITION??
- HARTREE APPROX:  $N$ -SYSTEM in  $\mathbb{R}^3$

## ■ QM HYPERREALITY

# EULER EQUATIONS

- AIR/WATER
- SMALL VISC HEAT CONDUCTIVITY ( $= 0$ )
- DENSITY  $\rho$
- MOMENTUM  $m = \rho u$
- VELOCITY  $u = (u_1, u_2, u_3) = \frac{m}{\rho}$
- INTERNAL ENERGY  $e$

# CONS: MASS, MOM, INT ENERGY

Find  $(\rho, m, e)$ :

$$\dot{\rho} + \nabla \cdot (\rho u) = 0$$

$$\dot{m} + \nabla \cdot (mu) + \nabla p = \text{force}$$

$$\dot{e} + \nabla \cdot (eu) + \gamma e \nabla \cdot u = \text{heatsource}$$

$$u \cdot n = 0 \quad \text{SLIP!!}$$

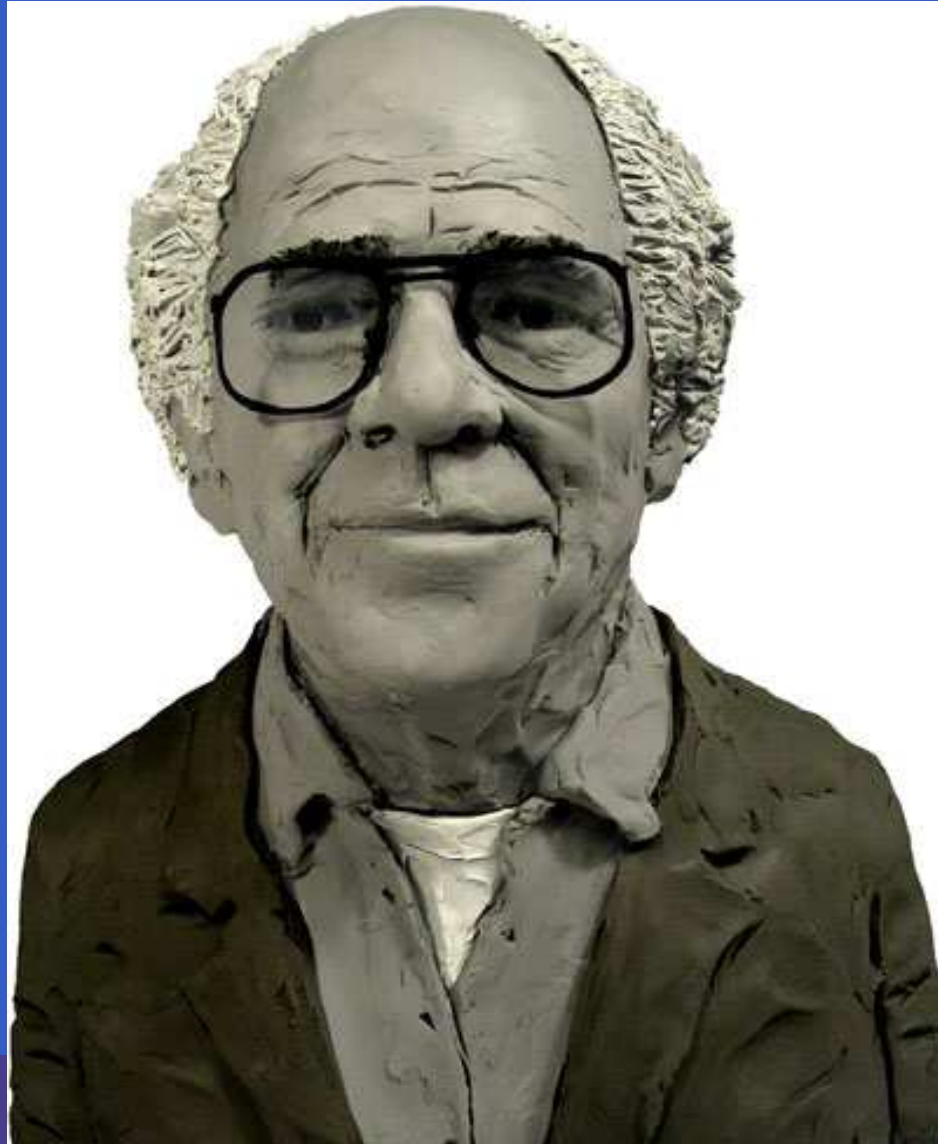
initial condition

- $\dot{v} = \frac{\partial v}{\partial t}$ ,  $\gamma > 0$  gas constant
- (INCOMPRESSIBLE)

# BAUDRILLARD (1929-2007)

- REAL = WHAT CAN BE SIMULATED
- HYPER-REAL = WHAT IS SIMULATED
- SIMULATION of NON-EXIST REALITY
- MODELS of REAL without REAL ORIGIN
- MASKS NON-EXIST of REAL REALITY

# SIMULATION of BAUDRILLARD



# 1ST-2ND ORDER SIMULATION

- BORGES
- EXACTITUDE in SCIENCE
- MAP COVERS TERRITORY

# 3RD ORDER SIM: HYPERREAL

- MAP REPLACES TERRITORY
- OUTSIDE REALM of GOOD and EVIL
- ONLY PERFORMATIVITY COUNTS
- CONTROL

# BANK ROBBERY: GOOD-EVIL

- REAL: PUNISHED for BEING REAL
- SIMULATED: NOT PUNISHED for being SIMULATION
- SIMULATED: PUNISHED for UPSETTING JUDICIARY SYSTEM

# DISNEYLAND

- IMAGE of
- AMERICAN SOC NEVER EXISTING
- MASKS NON-EXIST of REAL REALITY
- REPLACES REAL
- MODELS of WANTED REALITY

# WATERGATE PROCESS

- MASKS NON-EXISTENCE of
- NON-CORRUPT AMERICAN SOC
- SCANDAL:
- NOT BREAK-IN
- NOT COVER-UP of BREAK-IN
- RETURN to ORDER: FORD replaces NIXON
- ILLUSION of NON-CORRUPT SYSTEM

# MAGRITTE



# BARBIE DOLL



# The GULF WAR did not take place

- CNN REPORTERS WATCH CNN NEWS
- CNNs WAR DID NOT HAPPEN
- NEWS GENERATED by NEWS

# BAUDRILLARD

- REAL:
- Why is there SOMETHING, rather than NOTHING?
- THE REAL NO LONGER EXISTS
- HYPERREAL:
- Why is there NOTHING, rather than SOMETHING?

# DELEUZE (1925-1995)

- HYPER-REALITY:
- the ONLY REALITY there is

# REFLECTIONS of DELEUZE



# MODERN vs POSTMODERN

- MODERN: OBJ EXIST REAL WORLD
- POST-MODERN:
- HYPERREAL SIMULACRA of
- NON-EXIST REAL WORLD

# SECOND LIFE

- COMPUTER GAMES
- DOCU-SOAP
- PORNOGRAPHY

# BIBLE

- God created man in His own image, in the image of God He created him; male and female He created them.
- EXISTENCE of GOD?
- HUMAN BEING HYPERREAL?

# DIJKSTRA

- Originally I viewed it as the function of the abstract machine to provide a truthful picture of the physical reality. Later, however, I learned to consider the abstract machine as the *true* one, because that is the only one we can *think*; it is the physical machine's purpose to supply a *working model*, a (hopefully) sufficiently accurate physical simulation of the true, abstract machine.

# HYPERREAL PHYSICS

- SPACE-TIME
- STATISTICAL MECHANICS
- QUANTUM MECHANICS

# HYPERREAL PHYSICS

- SIMULATION of
- NON-EXISTING PHYSICS
- APPROXIMATIONS of
- NON-EXISTING EXACT SOLUTIONS

# HYPERREAL SOCIETY

- APPROXIMATION of
- NON-EXISTING PERFECT SOCIETY



# CLAY INST \$1 MILLION PRIZE

- (I) EXISTENCE OR
- (II) BLOWUP of
- EXACT SOLUTIONS
- EULER/NAVIER-STOKES EQUATIONS?
- WILL SHOW (II) BLOWUP
- NON-EXISTENCE OF EXACT SOLUTIONS
- HYPERREALITY

# LIN EQ: $v = u - \bar{u}$ WELLPOS?

$$\begin{aligned}
 \dot{v} + (u \cdot \nabla)v + (v \cdot \nabla)\bar{u} + \nabla q &= f - \bar{f} && \text{in } \Omega \times J \\
 \nabla \cdot v &= 0 && \text{in } \Omega \times J \\
 v \cdot n &= g - \bar{g} && \text{on } \Gamma \times J \\
 v(\cdot, 0) &= u^0 - \bar{u}^0 && \text{in } \Omega.
 \end{aligned}$$

(1)

- CONVECTION  $u$  REACTION  $\nabla \bar{u}$
- TRACE  $\nabla \bar{u} = \nabla \cdot \bar{u} = 0$
- UNSTABLE/STABLE EIGENVALUES
- EXPONENTIALLY UNSTABLE POINTW.

# VORTICITY EQUATION

$$\dot{\omega} + (u \cdot \nabla)\omega - (\omega \cdot \nabla)u = \nabla \times f \quad \text{in } \Omega, \quad (2)$$

■ EXPONENTIALLY UNSTABLE POINTW

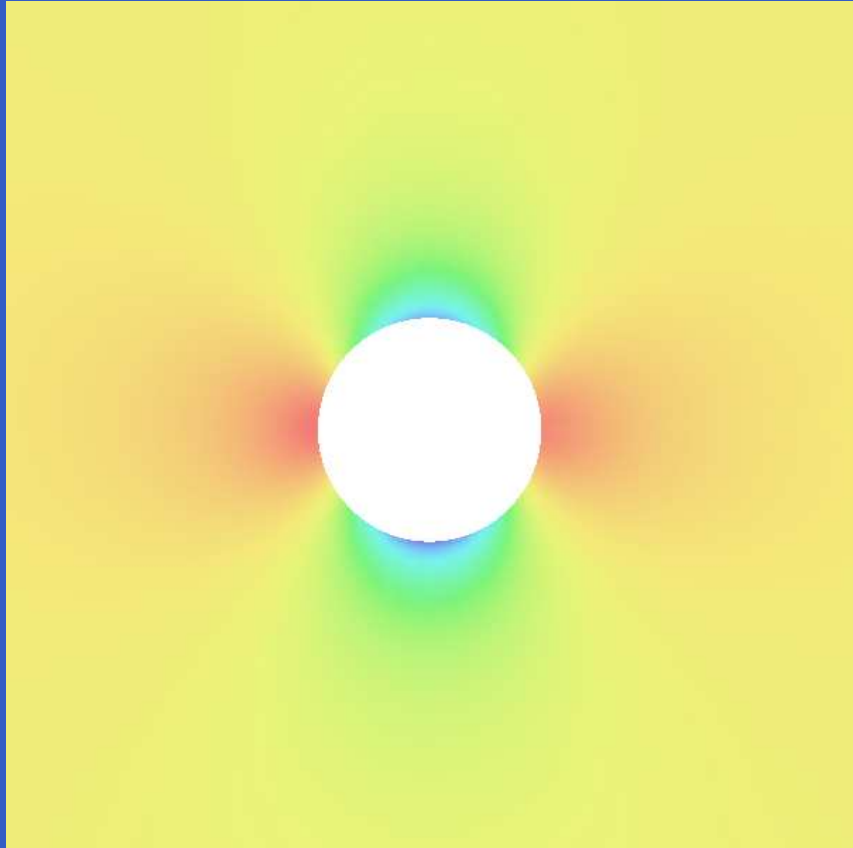
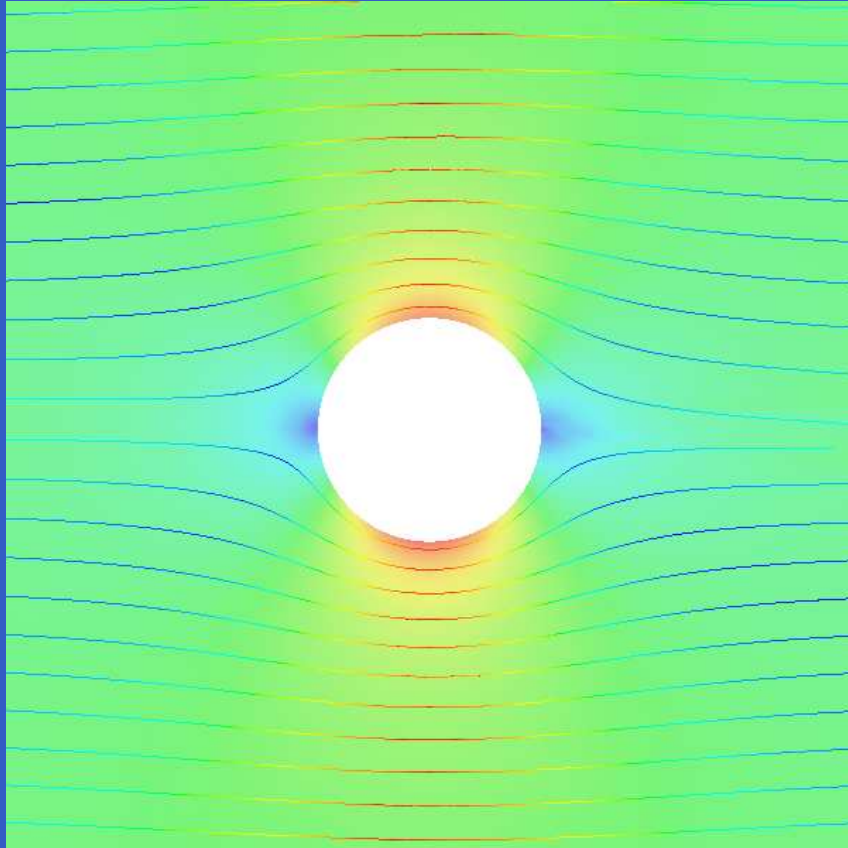
# EG2 EULER GENERAL GALERKIN

- RESIDUAL LS STAB GALERKIN:  $hR^2$
- MESH SIZE  $h$
- SLIP BC
- NO PARAMETER (VISC = 0)

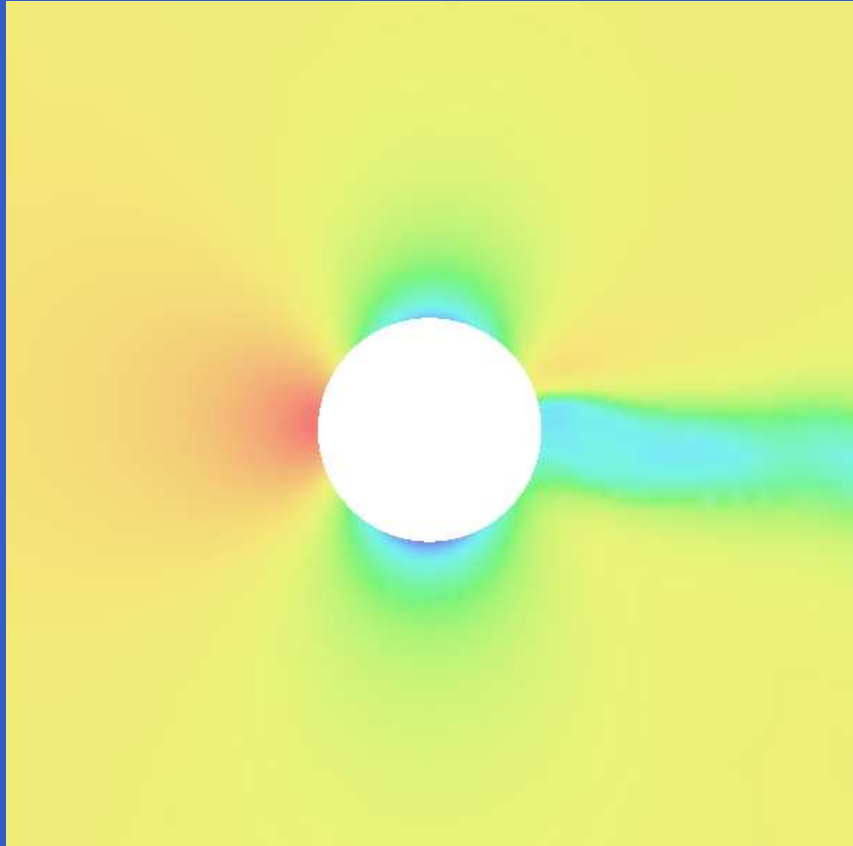
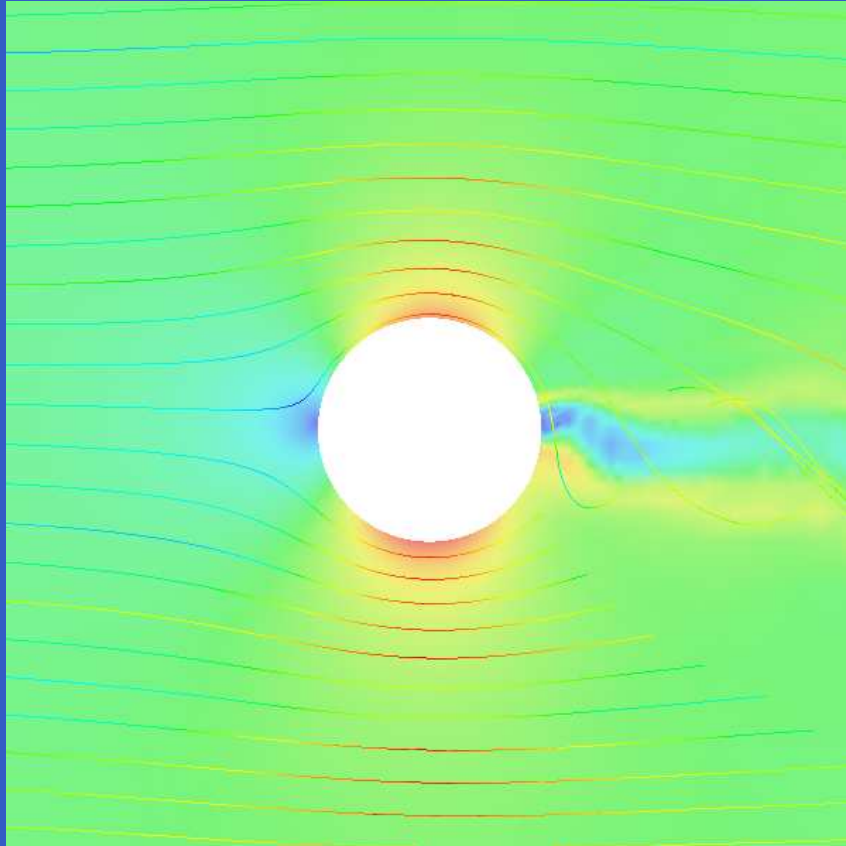
$$K(T) + D_h(T) = K(0), \quad K(T) \quad \text{KIN ENERGY}$$

$$D_h(t) = \int_0^T \int_{\Omega} hR^2 dx dt \gg 0 \quad \text{TURB-NONSMOOTH}$$

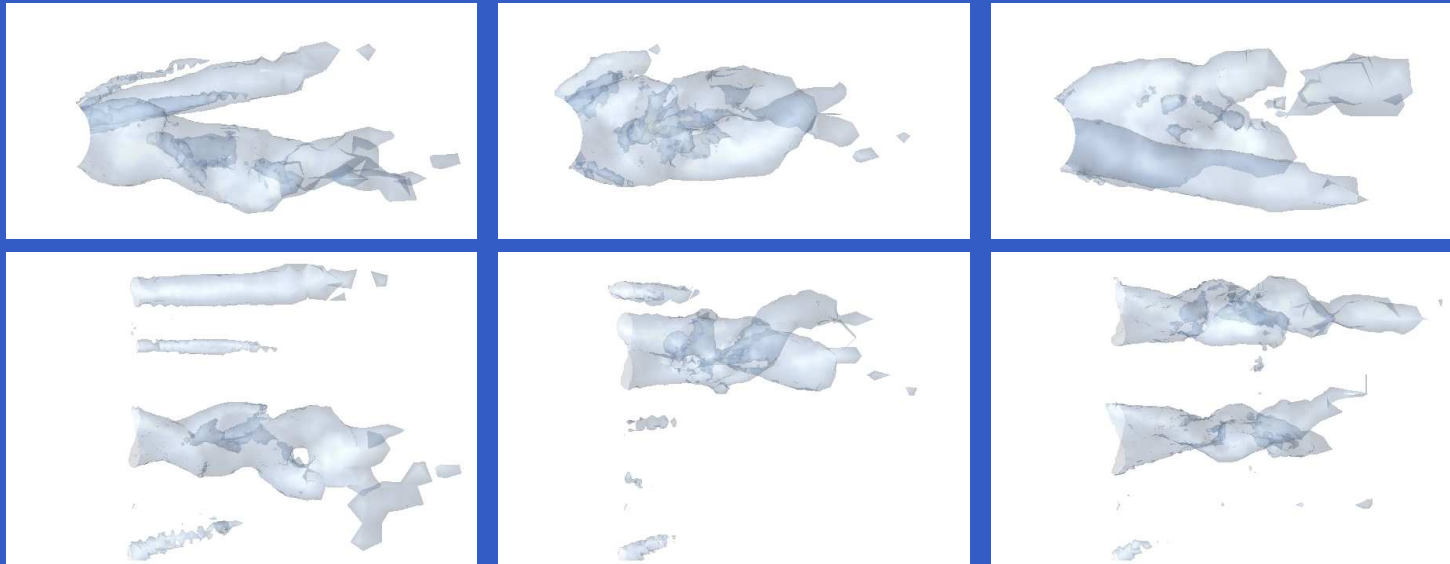
# POTENTIAL SOL DRAG = 0



# EG2 REAL SOL DRAG = 1

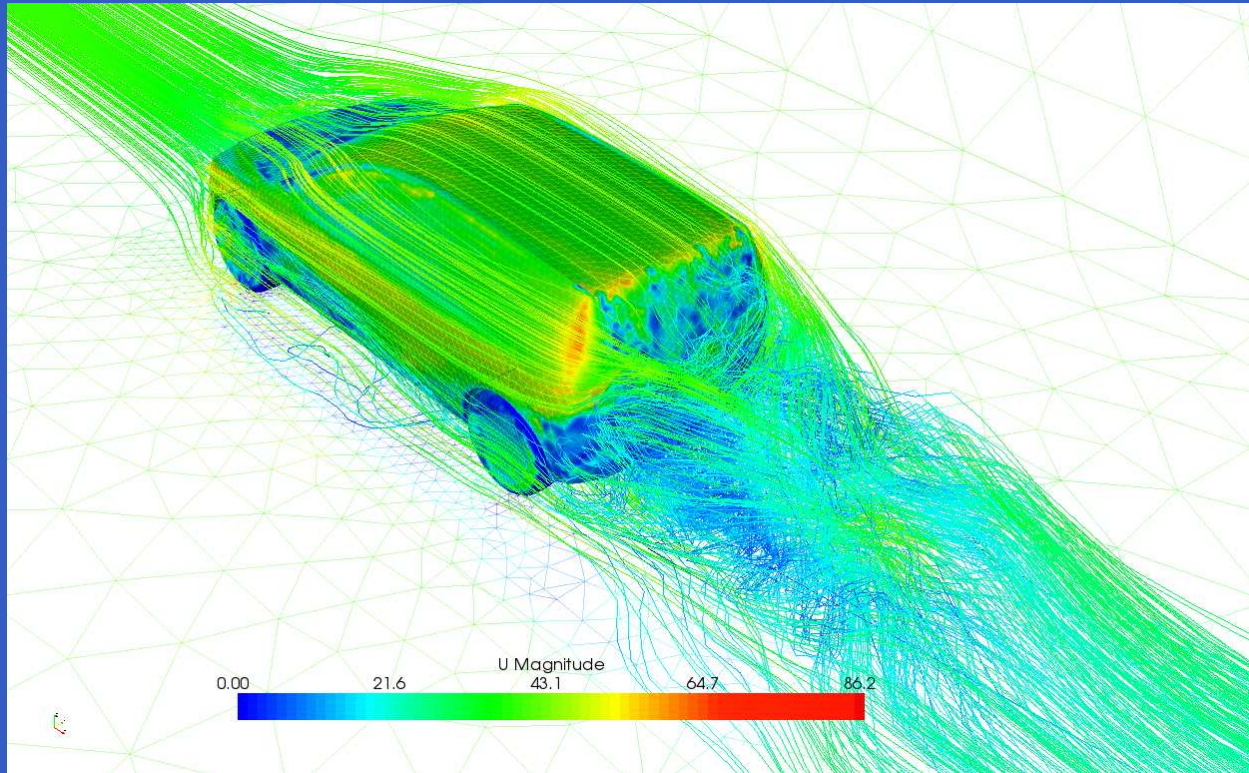


# VORTICITY



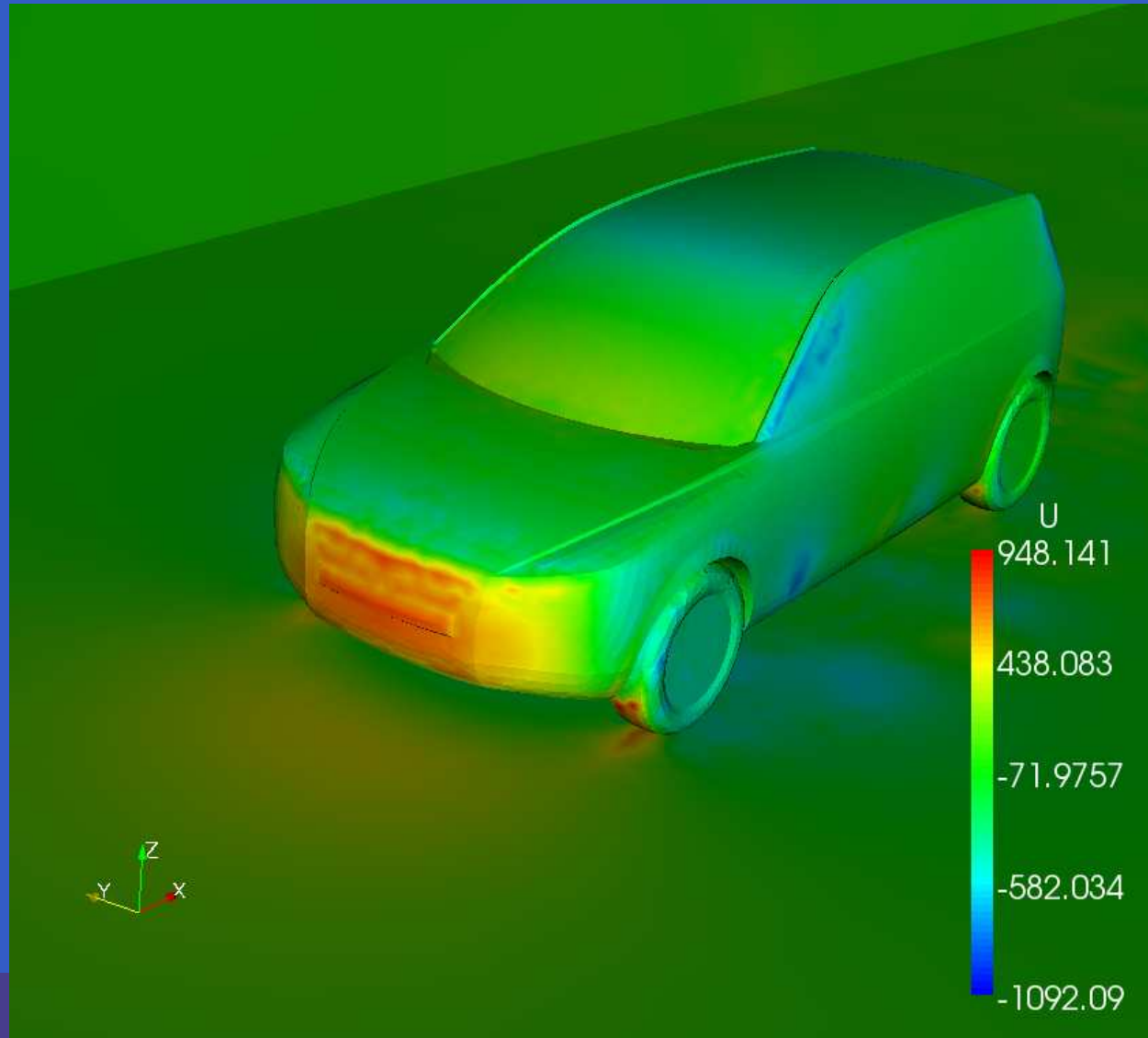
- EXPLAINS WHY IT IS POSSIBLE TO FLY: LIFT/DRAG
- EXACT POT SOLUTIONS DO NOT FLY!!
- COMP SOLUTIONS DO FLY!!

# VOLVO CAR

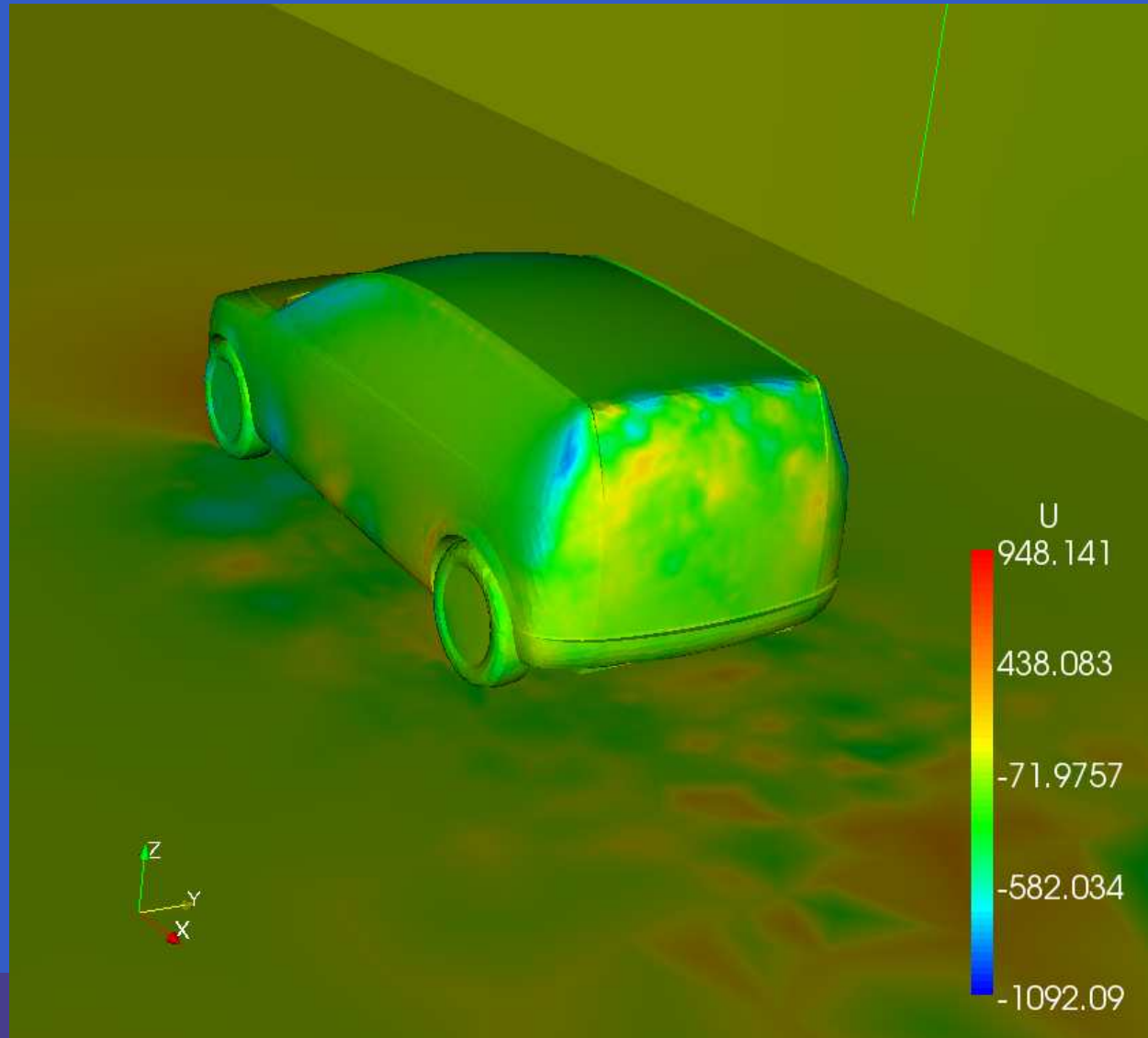


- DRAG  $\approx 0.33$ .
- VIRTUAL WIND TUNNEL

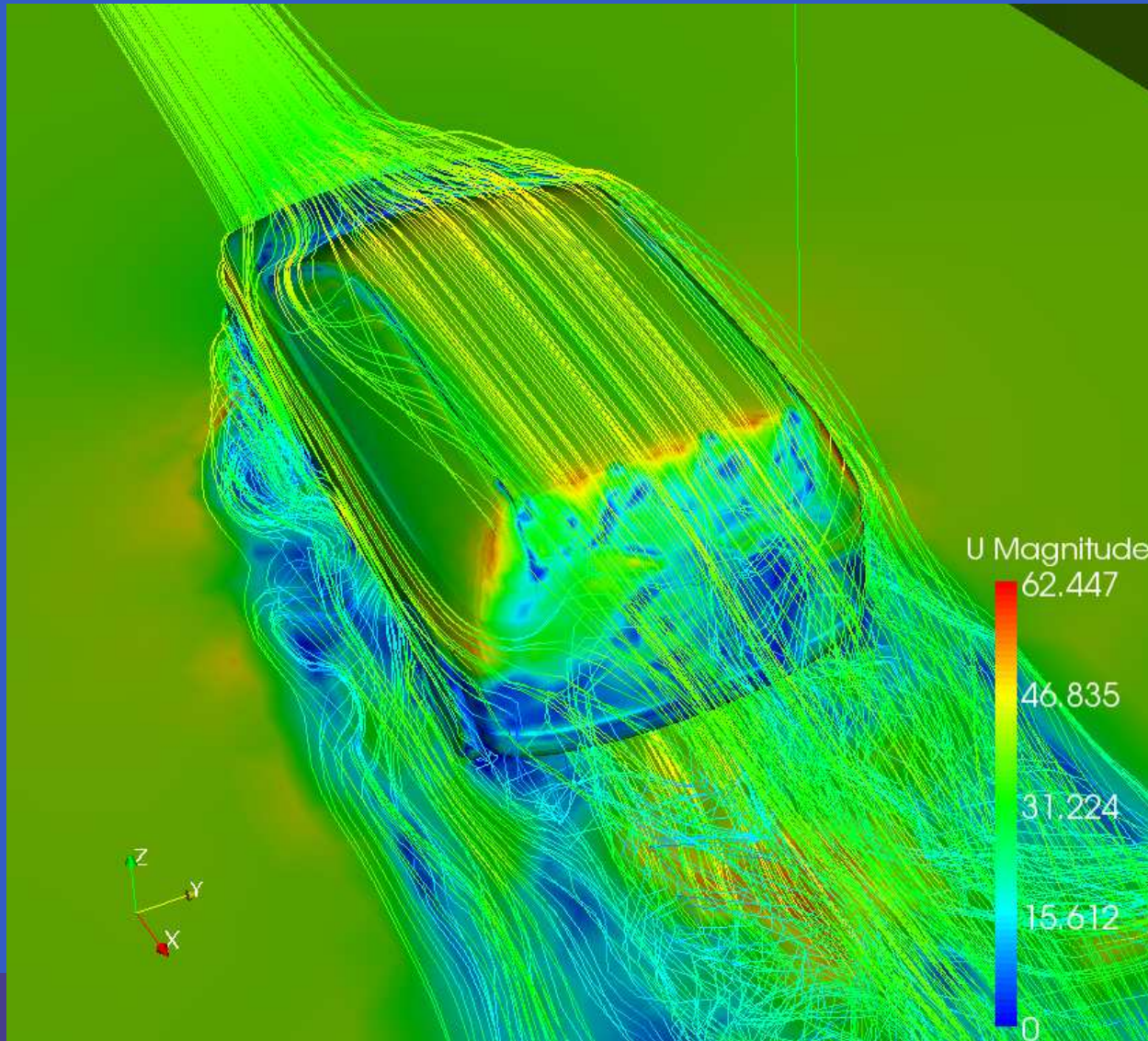
# CAR: PRESSURE



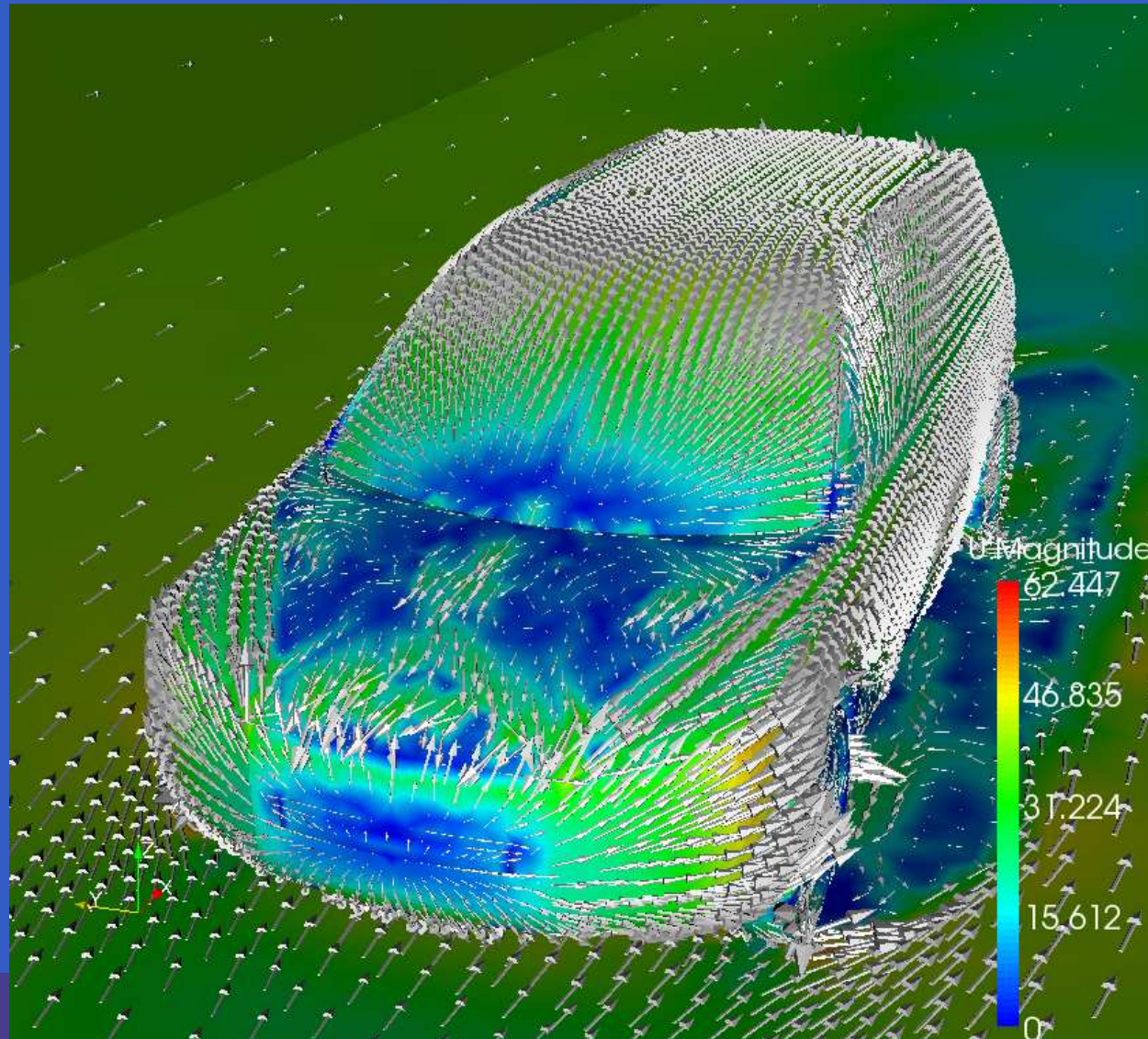
# CAR: PRESSURE



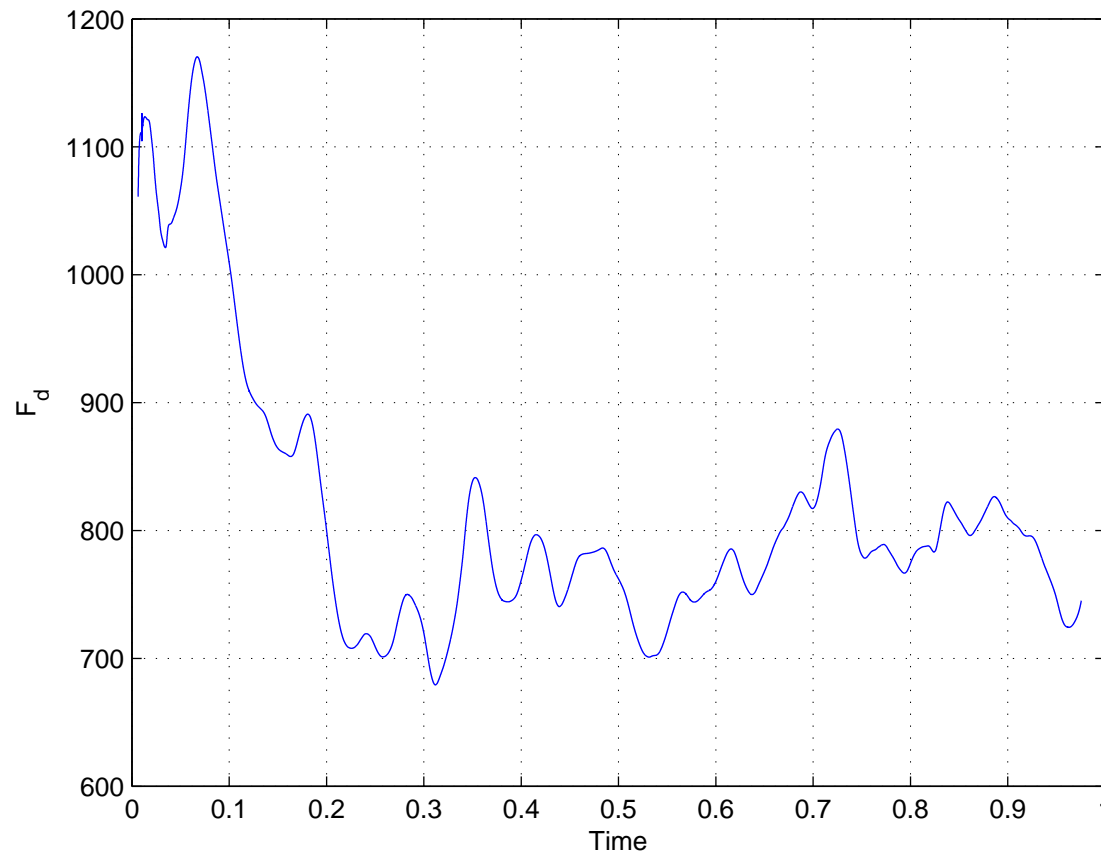
# CAR: VELOCITY



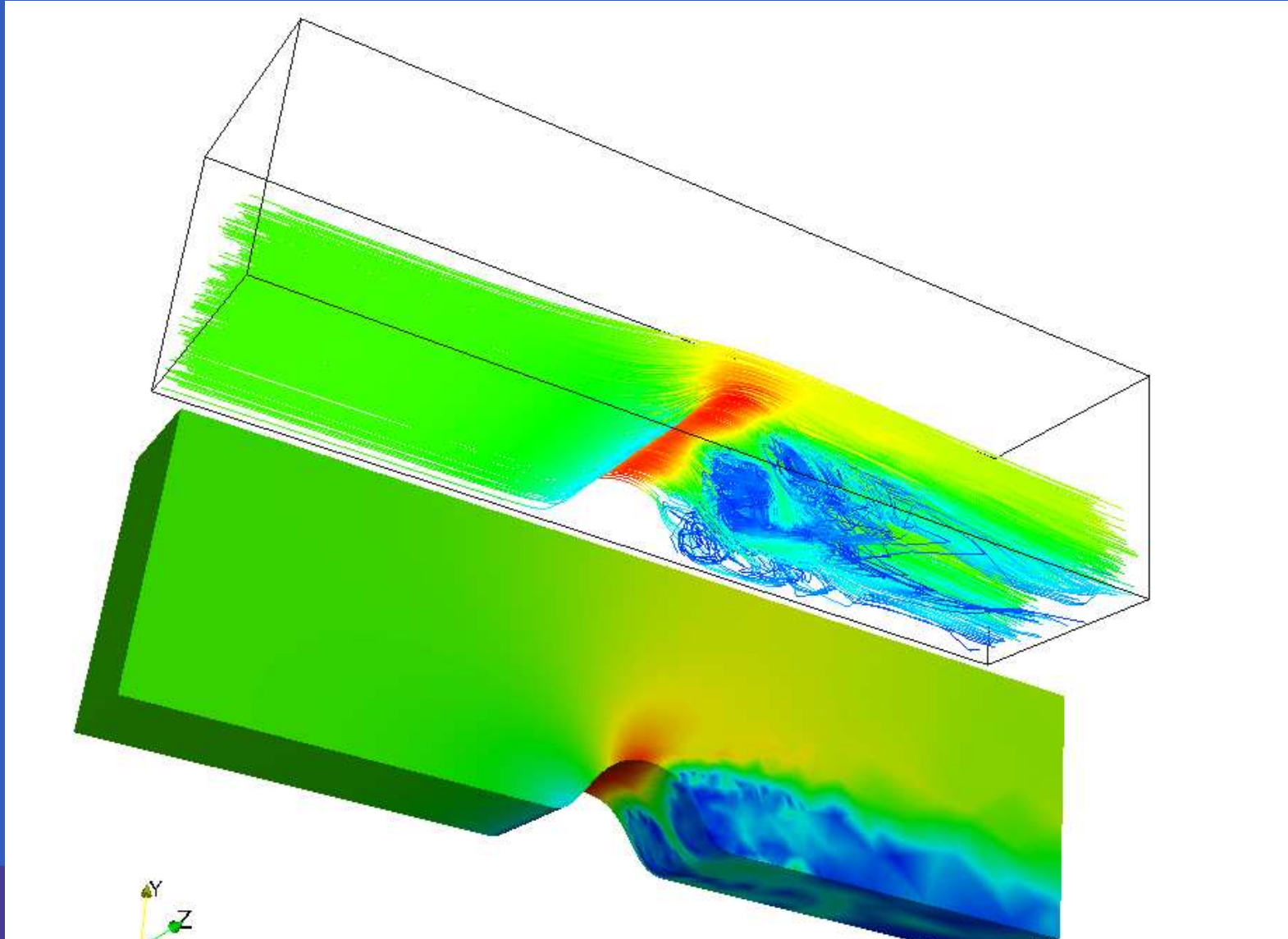
# CAR: VELOCITY



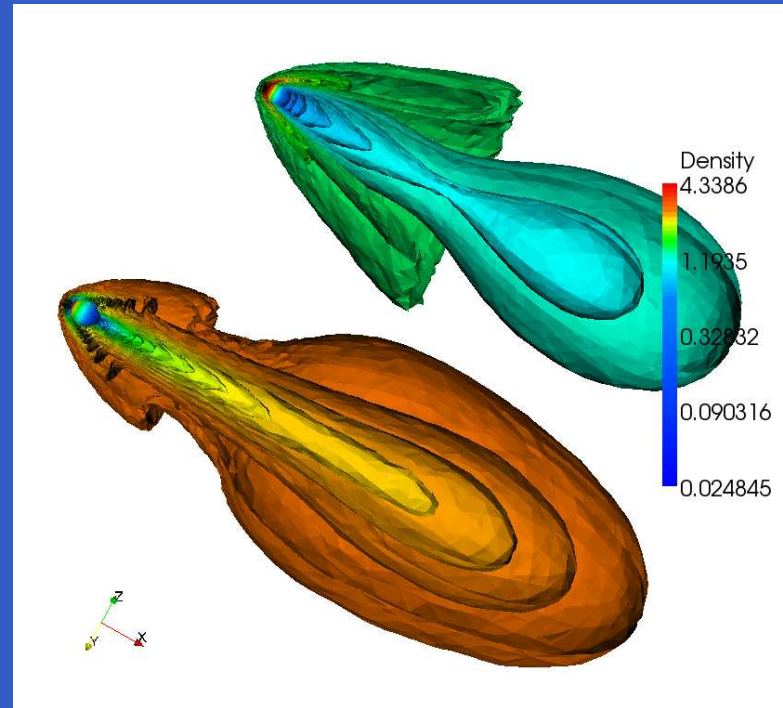
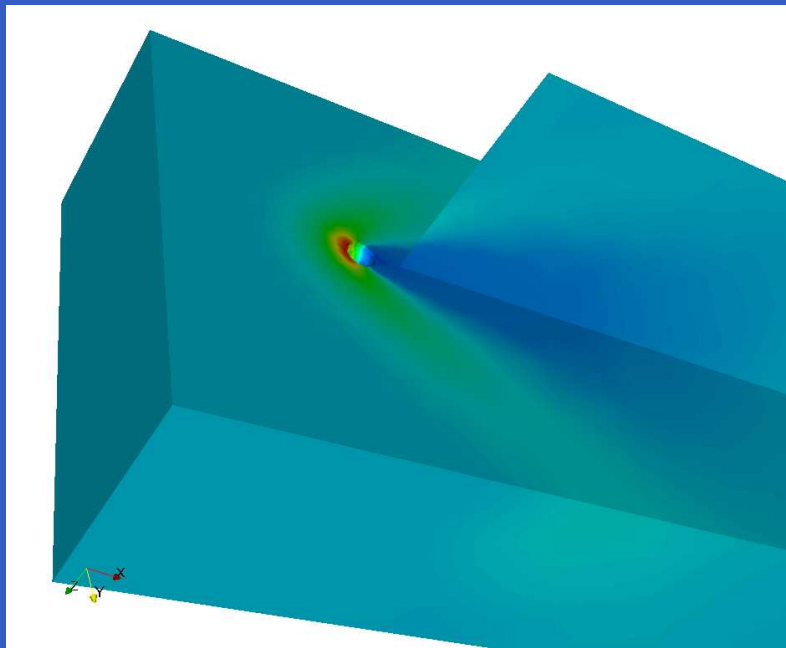
# CAR: DRAG



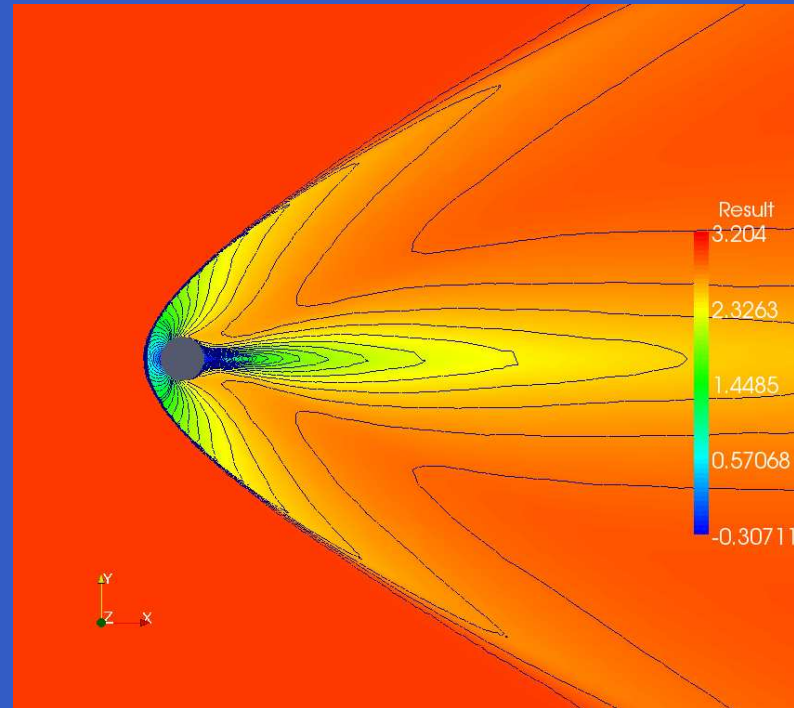
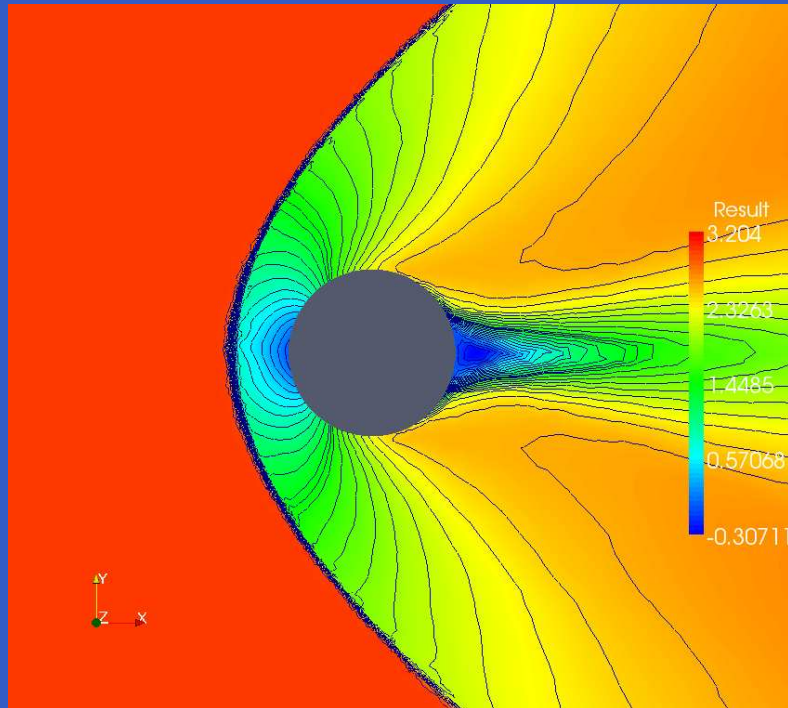
# HILL SEPARATION



# COMPRESSIBLE EULER SPHERE



# COMPRESSIBLE EULER 2D CYL



# PROOF OF BLOWUP

(A) EG2 WELLPOSED DRAG:  $H^{-1}$  PERTURB

(B) POT SOL NOT WELLPOSED

(C) EG2 TURB NONSMOOTH:

$$\|R\|_{-1} \approx h^{1/2} \quad \|R\|_0 \approx h^{-1/2}$$

(D) FINITE MESH SIZE ENOUGH

- $\rightarrow$  ONE MILLION DOLLAR

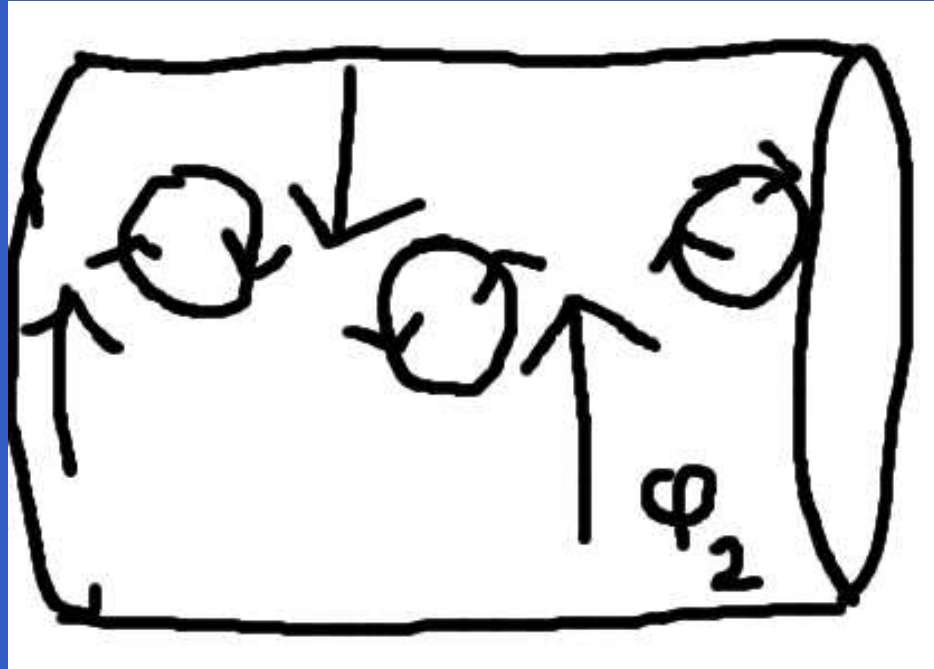
# EG2 WELLPOSED

- COMPUTED  $S$  MODERATE SIZE!!
- EG2 SOL REPRESENTATIVE SOLUTION!!
- EG2 BLOWUP  $\rightarrow$  BLOWUP
- NEW FACT:  $S$  MODERATE SIZE
- $S$  SOLVING DUAL LIN EQ:  
SMOOTH DATA CANCELLATION!!
- SOLVES D'ALEMBERT'S PARADOX = \$\$\$?

# BLOWUP REAR SEPARATION

- POT FLOW in half-plane  $\{x_1 > 0\}$ :
- $u = (x_1, -x_2, 0)$
- LIN EQ:  $\dot{\varphi}_2 - \varphi_2 = f_2$  RETARD
- $f_2 = f_2(x_3)$  oscill residual perturb
- $\varphi_2(t, x_3) = t \exp(t) f_2(x_3)$
- $\omega_1$ -vorticity:  $\dot{\omega}_1 + x_1 \frac{\partial \omega_1}{\partial x_1} - \omega_1 = 0$ , ACC
- INFLOW BC  $\omega_1(\bar{x}_1, x_2, x_3) = \frac{\partial v_2}{\partial x_3} = t \exp(t) \frac{\partial f_2}{\partial x_3}$ .
- Double exp growth  $\exp(t) \Rightarrow$  BLOWUP

# REAR SEPARATION



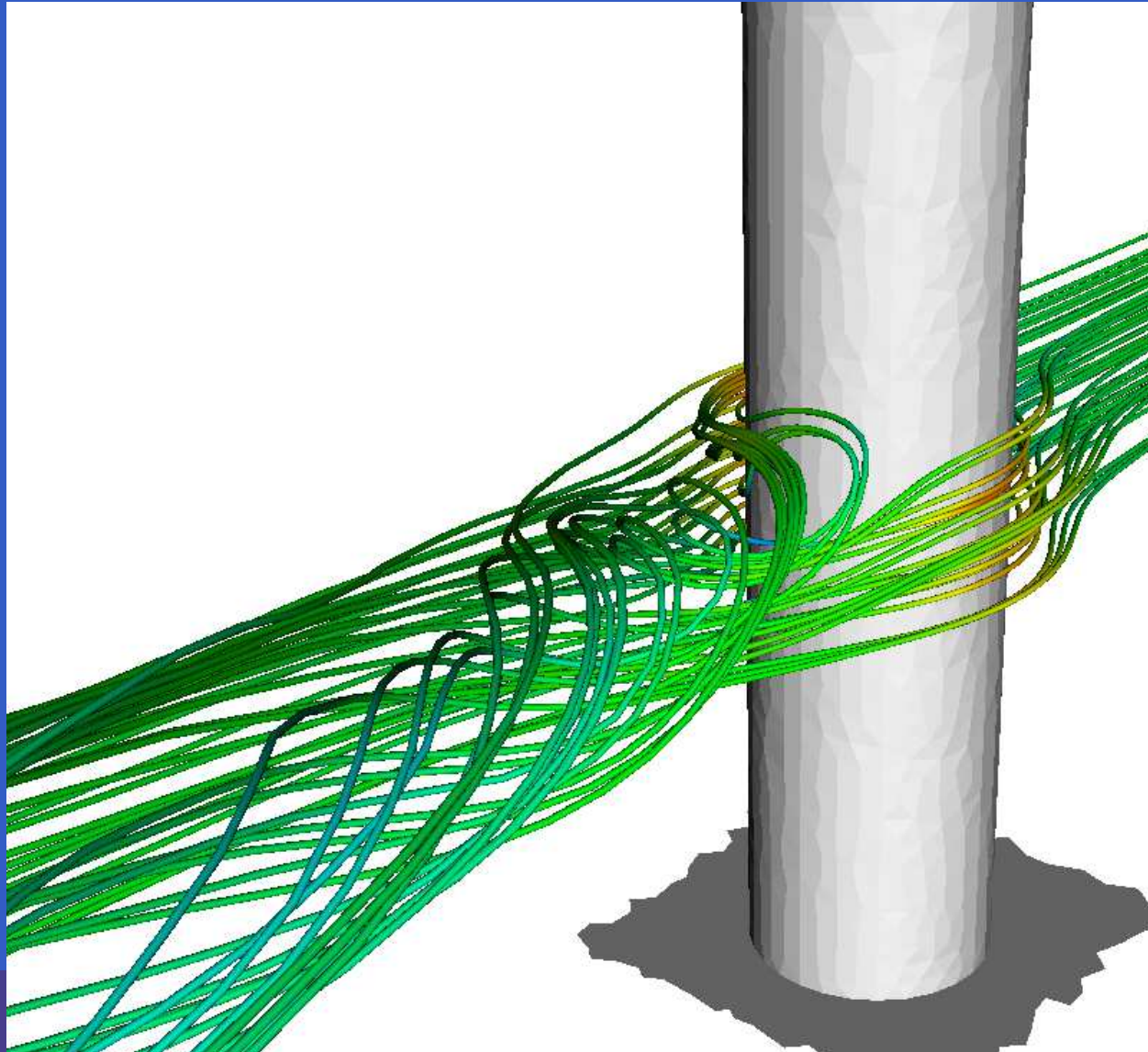
- Decentered opposing velocity
- generates streamwise  $\omega_1$  vorticity

# REAR SEPARATION

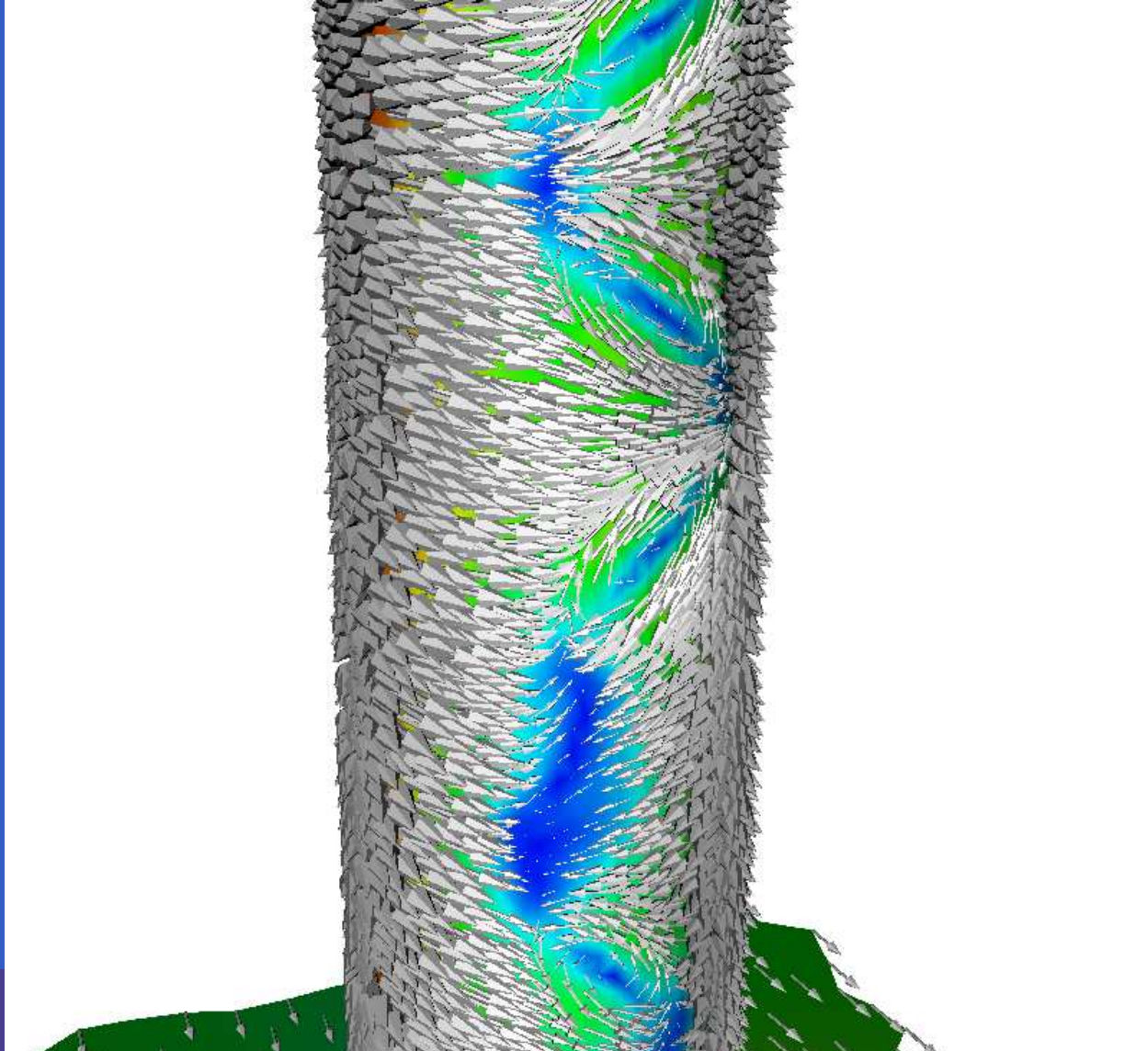


- Vortex stretching
- Streamwise vorticity streaks
- Oscillating diagonal pattern

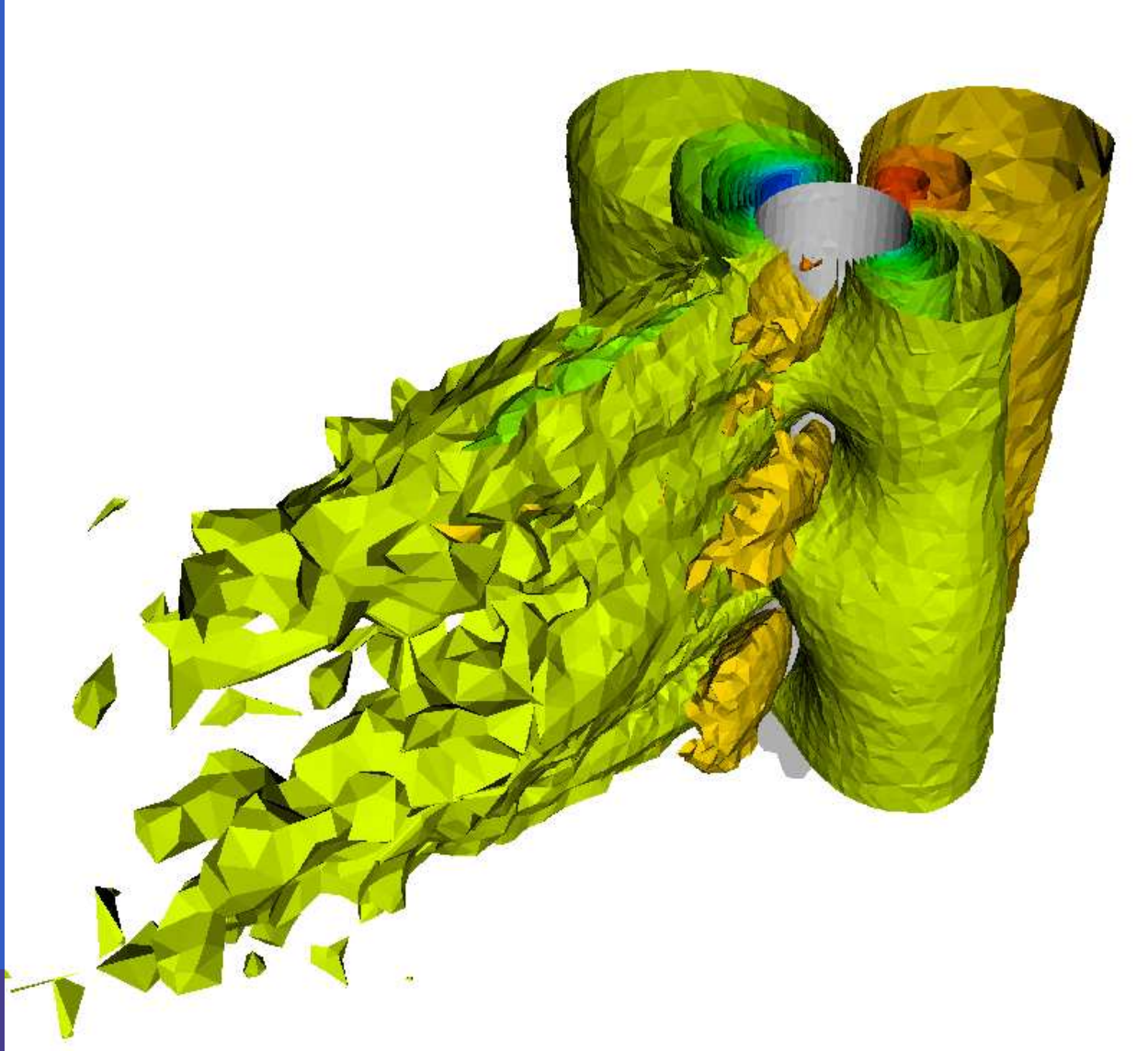
# STREAMLINES



# VELOCITY SURFACE



# PRESSURE LEVEL SURFACES



# CLASSICAL SEPARATION

- PRANDTL: ADVERSE  $\nabla P$ :  $\frac{\partial P}{\partial \tau} < 0$
- TANGENT VEL = 0  $\Rightarrow$  NORMAL  $\nabla P$
- $= \frac{\partial P}{\partial n} = 0$
- DOES NOT STICK
- WRONG FOR LARGE REYNOLDS!!

# NON-SEPARATION

$$\frac{\partial P}{\partial n} = \frac{U^2}{R}$$

- SLIP  $\Rightarrow \frac{\partial p}{\partial n} > 0$
- STICK: NON-SEPARATION
- TURBULENT BL  $\approx$  SLIP/FRICTION
- EXPLAINS DRAG/LIFT FLYING
- NO TURBULENCE – NO BIRDS/AIRPLANES

# CONCLUSION

- WELLPOSED COMP SOL BLOWUP = TURBULENT
- FINITE MESH SIZE SUFFICIENT
- HYPERREALITY REPLACING NON-EXIST REALITY
- SOLVES CLAY PROBLEM
- PUBLISHED IN BIT/JMFM